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ON THE STABILITY OF PLASMA IN A NONUNIFORM MAGNETIC FIELD AND ON THE MECHANISM OF SOLAR FLARES*

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ABSTRACT

This is a study of the state of magnetohydrostatic equilibrium in a plasma with a frozen-in nonuniform magnetic field. It is shown, that contrary to the conclusion reached in [1, 2], such a state is stable. As a consequence, the original case stemming from the theory of chromospheric flares in the Sun developed in references [1 and 2], is found to be incorrect.

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COVER TO COVER TRANSLATION

Assertion was made in the works [1, 2] of current layer instability in the plasma, leading to its spontaneous compression near the neutral point (point where the magnetic field intensity is zero). This assertion lies at the basis of the theory of chromospheric flares on the Sun, developed in [1, 2] and also in [3], where it is confounded at many places with the idea of the pinch-effect.

These papers consider the one-dimensional problem of plasma behavior when it carries a frozen-in magnetic field

* K voprosu o neustoychivosti plazmy v neodnorodnom magnitnom pole i o mekhanizme solnechnykh vspyshek.

thus when the conductivity $\delta = \infty$, under conditions when the magnetic pressure gradient tends to compress the plasma. The conclusion of instability is made on the basis of the fact that at a uniform adiabatic compressiom the magnetic pressure $H^2/8\pi$ increases as ρ^2 (ρ being the density of the plasma), whereas gas pressure p is proportional to ρ^γ, with γ always < 2.

However, the difference in the laws of pressure increase cannot in itself be the cause of spontaneous compression, for the dynamic behavior of the medium is not simply determined by pressure, but by its gradient. That is why the reasonings brought out in [1, 2], are apparently based on the assumption that the plasma-compressing magnetic pressure gradient must also increase by ρ^2/ρ^γ times faster than the counteractive compression of the gas pressure gradient, and the equilibrium condition

$$\frac{\partial}{\partial x}\left(p + \frac{H^2}{8\pi}\right) = 0 \qquad (1)$$

may then be only satisfied at random. This assumption is incorrect.

The variation of pressure gradient (either magnetic or gas) is not simply proportional to its initial value, but depends also on the pressure value itself: in those regions, where the main contribution to total pressure is made by the plasma, the gas pressure gradient will rise more rapidly than the pressure itself, and vice versa. This is easy to prove, considering for instance the variation of magnetic pressure gradient at compression with a velocity v. Utilizing thr freeze-in condition (see for example [4])

$$\frac{\partial H}{\partial t} + v\frac{\partial H}{\partial x} = -H\frac{\partial v}{\partial x}, \qquad (2)$$

we find

$$\frac{d}{dt}\left(\frac{\partial H^2}{\partial x}\right) \equiv \left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x}\right)\frac{\partial H^2}{\partial x} = -3\frac{\partial H^2}{\partial x}\frac{\partial v}{\partial x} - 2H^2\frac{\partial^2 v}{\partial x^2}. \qquad (3)$$

The last term of the right-hand part of the expression (3) means exactly that gradient variation of magnetic pressure at a fixed point of the plasma depends on the pressure value itself too. Naturally, a similar result takes also place for gas pressure. At the same time, to preserve the equilibrium, compression must be greater in those regions where the pressure is determined mainly by the plasma, and smaller — where the main role belongs to magnetic pressure.

This effect was not taken into account in works [1, 2]; the gas and the magnetic pressures were substituted by their mean values in the whole region, and the compression was presumed uniform*. This is what led to the erroneous deduction of unsteadt state satisfying the equation (1).

One may determine the true character of the compression and convince himself of the absence of any instabilities with the aid of the standard method of small perturbations. The problem is defined by the following equations (in the Lagrange form):

$$\rho_0 \frac{\partial^2 x}{\partial t^2} = -\frac{\partial}{\partial a}\left(p + \frac{H^2}{8\pi}\right), \; \rho \frac{\partial x}{\partial a} = \rho_0, \; \frac{\partial}{\partial t}\left(\frac{H}{\rho}\right) = 0, \; \frac{\partial}{\partial t}\left(\frac{p}{\rho^\gamma}\right) = 0, \quad (4)$$

where $x(a, t) = a + \xi(a, t)$ is the coordinate of the material point, a is its initial value at $t = 0$. Let us consider that in the initial state ($\xi = 0$, $p = p_0(a)$, $\rho = \rho_0(a)$, $H = H_0(a)$) the system is in equilibrium and passes, as a result of small perturbation into the state $p = p_0(a) + p'(a, t)$, $\rho = \rho_0(a) + \rho'(a, t)$, $H = H_0(a) + H'(a, t)$, where the displacement ξ and the quantities designates by are considered small. If we neglect the quantities of the second order of smallness, equations (4) may be transformed into the following form:

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* The assumption about a uniform (self-modeled) compression is also found in the work [5]. However, in view of the nonuniformity of the considered medium, the assumption of self-modelness is inapplicable.

$$\rho_0 \frac{\partial^2 \xi}{\partial t^2} = \frac{\partial}{\partial a}\left\{\left(\gamma p_0 + \frac{H_0^2}{4\pi}\right)\frac{\partial \xi}{\partial a}\right\}, \tag{5}$$

$$\rho' = -\rho_0 \frac{\partial \xi}{\partial a}, \quad p' = \gamma \frac{p_0}{\rho_0}\rho', \quad H' = \frac{H_0}{\rho_0}\rho',$$

Since the coefficients of these linear equations do not depend on time, the solutions may be sought in the form

$$\xi\,(a, t) = \zeta\,(a)\, e^{i\omega t}. \tag{6}$$

At the same time, we find from equations (5)

$$-\omega^2 \rho_0 \zeta = \frac{\partial}{\partial a}\left\{\left(\gamma p_0 + \frac{H_0^2}{4\pi}\right)\frac{\partial \zeta}{\partial a}\right\}. \tag{7}$$

Multiplying both parts of the equation (7) by and integrating along the whole range of values a, we shall obtain

$$-\omega^2 \int_{-\infty}^{+\infty} \rho_0 \zeta^2 da = -\int_{-\infty}^{+\infty} \left(\gamma p_0 + \frac{H_0^2}{4\pi}\right)\left(\frac{\partial \zeta}{\partial a}\right)^2 da + \left(\gamma p_0 + \frac{H_0^2}{4\pi}\right)\zeta \frac{\partial \zeta}{\partial a}\Big|_{-\infty}^{+\infty}. \tag{8}$$

Since at $a = \pm\infty$ the deformation may be deemed absent, the last term in the right-hand part of the expression (8) becomes zero and it follows from the positiveness of subintegral expressions that

$$\omega^2 \geqslant 0. \tag{9}$$

Therefore, the problem does not admit perturbations that would lead to instability relative to spontaneous compression (and also rarefaction), and this is in contradiction with the conclusion arrived at in the work [1, 2].

The case $\omega = 0$ corresponds to the condition $\partial^2\xi/\partial t^2 = 0$. as may be seen from the expression (8), i. e. it corresponds to the shift of all the system as a whole. In the remaining cases all perturbations propagate in the form of waves (magnetoacoustic in the given case) with an amplitude dependent on the coordinate a, as this must be in a spatially-nonuniform medium.

When the perturbation is adiabatically slow, we may neglect the acceleration of the mediuj, i. e. we may consider that $\partial\zeta/\partial a = 0$, In this case the first of the equations (5) defines the law of compression .

$$\frac{\partial\xi}{\partial a} = \frac{C}{\gamma p_0(a) + \frac{1}{4\pi} H_0^2(a)}, \qquad (10)$$

for which the equilibrium condition is fulfilled at every stage. The constant C is determined by the value of $\partial\xi/\partial a$ for a certain value $a = a_0$.

As this follows from the expression (10) and from the condition $P_0 = p_0(a) + \frac{1}{8\pi} H_0^2(a) = \mathrm{const}$ (see (1), P_0 is the total pressure), the compression will be greater where gas pressure is relatively high, and smaller in those regions where magnetic pressure is high.

ADDENDUM AT CORRECTION *. In order to better illustrate the substance of the above critical observations, I shall take the liberty to bring forth a simple example. In the work by Severnyy referred-to above, an essentially nonunoform medium is considered, the magnetic field contributing in its various portions to the total elasticity of the system in different fashions. Such system is entirely analogus to spring with a nonuniform hardness. It is quite obvious that a nonuniform spring will compress irregularly : it will do so more where hardness is lesser and vice versa. However, in the works by Severnyy referred-to, the compression is assumed to be uniform in advance, while the disruption of equilibrium as a result of that artificial assumption is interpreted as the instability of the considered system. The errors contained in

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* I have been made aware that this note shall be published simultaneously with Severnyy's response. I am not acquainted with the tenor of his paper./(see NASA TT F-8377)/

the works [1 — 3, 5] reside precisely there.

*** THE END ***

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